CONVERGENCE OF NUMERICAL SOLUTIONS OF OPEN-ENDED WAVEGUIDE BY MODAL ANALYSIS AND HYBRID MODAL-SPECTRAL TECHNIQUES.

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ABSTRACT

Two different methods are considered to deal with open-ended waveguides with infinite metallic flange. The first one, Modal Analysis, is valid only when the aperture is radiating into a lossy media. The second one is based on a hybrid modal-spectral technique, and it is valid for any media with or without losses.

The rectangular aperture problem is solved by both methods, and the influence of different parameters on the convergence of numerical solutions is studied for each method.

Finally, a comparison between both methods is presented for lossy and low-loss medias.

INTRODUCTION

Open-ended waveguides are very useful for many applications such as feed for reflector antennas, flush-mounted antenna for space-craft, or thermography and hyperthermia applicators.

The aperture problem has been solved by different approximate methods. Most of them consider only the fundamental mode in the aperture (1). Others (2), (3), (4) consider the fundamental and higher modes in the aperture in order to solve the problem more accurately.

When the open-ended waveguide is radiating into lossy medias, such as biological tissues or plasma, a model, proposed in references (4) and (5) will be considered here. In this model, the lossy media is assumed to be inside a large cross-section imaginary waveguide. The electromagnetic fields are concentrated near the aperture due to dielectric losses. Therefore, the effects produced by metallic walls of the large imaginary waveguide are negligible. With this assumption the aperture problem is reduced to a problem of discontinuity between two different waveguides with cross-sections S_i and S_0, being $S_0 \gg S_i$, Fig. 1. Then the problem can be solved by modal analysis (6), (7), assuming that the electromagnetic fields are expressed as a superposition of the eigen-modes in each region.

This model is not valid for low-loss or lossless medias. However, the angular spectrum of plane waves can be used to represent the electromagnetic field in half-space in any kind of media (8).

According to the above consideration, we propose a second method, that uses eigen-modes in the waveguide and angular plane-wave spectrum in half-space, see Fig. 2. Then, this method will be valid for any media with or without losses, and it will be called Hybrid Modal-Spectral Method (HMSM)

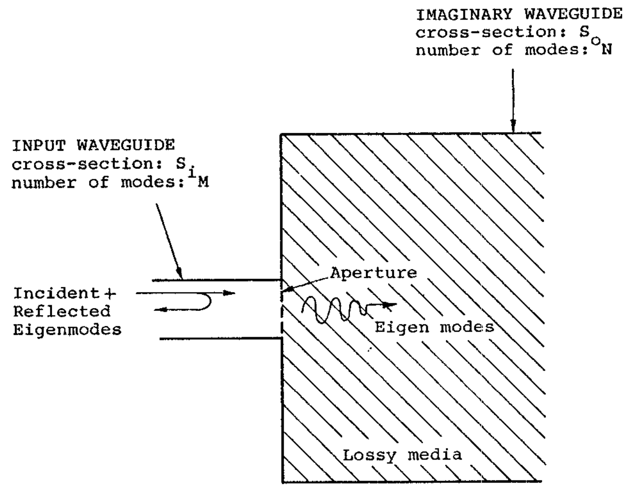

Fig. 1 Proposed model for an aperture radiating in lossy media.

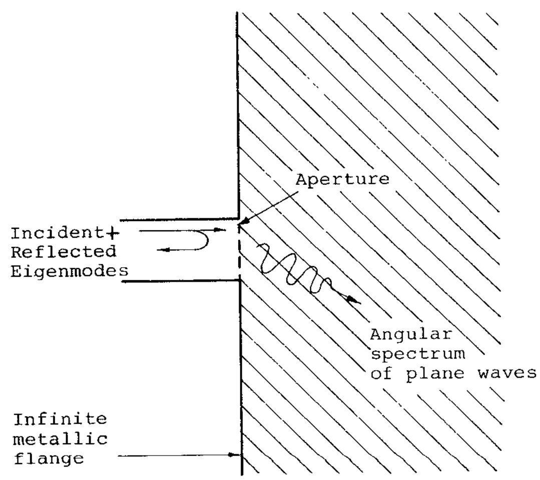

Fig. 2 Aperture radiating into half-space.

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Although the analysis is exact for both methods, there are some difficulties in the numerical resolution of the aperture problem.

First, in modal analysis, an imaginary waveguide of a large cross-section tries to simulate the open space for lossy medias, and the imaginary waveguide dimensions must increase when the losses decrease. Second, the number of modes retained in each waveguide is finite. Finally, when the problem is solved by HMSM the integration in half-space is performed numerically with a finite number of samples. These factors, i.e., imaginary waveguide dimensions, number of modes and number of samples, influence on the numerical solution of the aperture problem, and the convergence of the results must be studied.

Some results have been published about the influence of the ratio "number of modes/cross-section" for each waveguide on the numerical solutions of discontinuities found by modal analysis, (7) and (9), but in these works the two waveguides have a similar cross-section. In this paper the imaginary output waveguide simulates the open space, so its cross-section is very large (about 100 times the input waveguide cross-section); this leads to considering a very large number of modes in the output waveguide (from 400 to 6400) in order to solve the problem accurately, and the ratio between the number of modes in each waveguide, M/N, is more critical in order to save computer time and enhance accuracy.

The aperture is characterized by the reflection matrix S_{11}, where the coefficient S_{11} (i,j) is the amplitude of the ith reflected mode for jth incident mode, and then S_{11} (1,1) is the reflection coefficient for the fundamental mode (TE_{10}). The reflection matrix contains not only the reflection characteristics of the fundamental mode, but also those of the higher-order evanescent modes. This matrix is calculated by both methods and the results are then compared.

In the previous works the convergence has been studied by comparing the TE_{10} characteristics only. In this paper the convergence of the entire reflection matrix S_{11} is studied.

The electromagnetic fields at the aperture for any incident field in the waveguide are easily calculated from the reflection matrix S_{11}. When TE_{10} mode is incident, the aperture field obtained by both methods is plotted and then compared.

The reflection matrix S_{11}, characterizes the aperture and allows to join the aperture problem with other waveguides discontinuities described by their scattering matrices, as it has been proposed in (10). The scattering matrix of each discontinuity is obtained by means of modal analysis. So, modal analysis and HMSM can be applied together to design and optimize circuit components with multiple transverse discontinuities, such as adapters, corrugated polarizers, horns, etc, including the radiating aperture.

Finally, the radiation characteristics of the complete structure can be obtained with the proposed techniques.

MODAL ANALYSIS

Modal Analysis (6), (7) is a standard computer oriented method for solving discontinuity problems in waveguides.

The aperture is considered as a discontinuity between two waveguides of very different cross-section, see Fig. 1, and the electromagnetic fields are expressed as a superposition of the incident and reflected modes in each waveguide:

$$\vec{E}^{I,O} = \sum_i (d_i^{I,O} + a_i^{I,O})\, \vec{e}_i^{\,I,O}$$

$$\vec{H}^{I,O} = \sum_i (d_i^{I,O} - a_i^{I,O})\, \vec{h}_i^{\,I,O}$$

where: I,O means input and output waveguide respectively.

$\vec{e}_i$, $\vec{h}_i$ are the electric and magnetic fields for the i-mode.

d_i, a_i are the amplitudes of the incident and reflected i-mode.

The continuity condition on transverse components of the electromagnetic fields is applied at the discontinuity plane, and it leads to obtaining the whole scattering matrix of the aperture, S. The scattering submatrix S_{11}, is the reflection matrix mentioned before.

Numerical Results.

A rectangular aperture of cross-section S_i=AxB radiating into different biological lossy medias is considered. TE^x family modes are used to represent the fields in each region.

Sixteen modes have been retained in the input waveguide (M=16). It has been proved that this number of modes is enough to represent the electromagnetic field in the considered input waveguide (A=19 mm, B=19 mm).

The problem has been solved considering different number of modes in the imaginary output waveguide (N=400, 1600, 6400), and also different imaginary waveguide cross-sections (S_O=25, 100, 400 S_i).

The reflection coefficient for the TE_{10} mode $S_{11}(1,1)$, versus the parameters N and S_O, is shown in table I for three different lossy medias. Note that the ratio $(M/N)/(S_i/S_O)$ is maintained fixed in each diagonal.

From this table it can be observed that the reflection coefficient for the fundamental mode TE_{10} is not very dependent on the number of modes used in the output waveguide. Also the imaginary waveguide dimensions necessary to simulate the open space can be obtained for each lossy media. When $S_O \geqslant 25\ S_i$ (skin or muscle) or $S_O \geqslant 100\ S_i$ (fat or bone) the effect produced by metallic walls of the imaginary waveguide is negligible, and the model is valid.

From the comparison of the other reflection matrix coefficients $S_{11}(I,J)$ for different values of the parameters N and S_O, the following conclusions can be observed:

	Number of Modes M	N	5A x 5B $\|\rho\|$	5A x 5B ϕ	10A x 10B $\|\rho\|$	10A x 10B ϕ	20A x 20B $\|\rho\|$	20A x 20B ϕ
ε_r = 46-j13	16	400	.2076	171.9	.1907	172.0		
	16	1600	.2092	171.9	.2077	171.8		
ε_r = 5.8-j0.6	16	400	.7238	39.7	.6735	34.1	.6721	29.2
	16	1600	.7255	40.3	.6793	36.2	.6776	35.1
	16	6400	.7258	40.4	.6812	36.9	.6834	37.2
ε_r = 5.8-j0.06	16	400	.7673	38.3	.6066	28.7	.6756	25.4
	16	1600	.7688	38.9	.6137	30.9	.6813	31.4
	16	6400	.7691	39.0	.6160	31.6	.6874	33.5

Tabla I. TE_{10} reflection coefficient (magnitude≡$|\rho|$ phase≡ϕ) of a square aperture (AxB=19x19mm) for different lossy medias. Frequency=2 GHz. Permittivity in the waveguide ε_r=30.

- The reflection matrix coefficients S_{11} (I,J) for higher modes are very influenced by the number of modes, M and N, used in each waveguide, and the higher the order of the mode the greater the influence.
- If we change the cross-sections, S_i and S_0, and also the number of modes, M and N, but maintaining the ratio $(M/N)/(S_i/S_0)$ fixed, the reflection matrix will not change considerably.
- If $(M/N)\gg(S_i/S_0)$ the calculated reflection coefficient for the higher-order modes considered (i>>1, j>>1) is $S_{11}(i,j)\simeq 1$. This results can be explained by the fact that for theses modes the continuity condition at the aperture can not be satisfied with the number of modes retained in the output waveguide.

The magnitude of the transverse electric field at the aperture is plotted for the four cases shown in table I.a, see Fig. 3. For the two cases when $(M/N)=(S_i/S_0)$ the plottings are equal. For $(M/N)<(S_i/S_0)$ the aperture field is very similar to that of $(M/N)\neq(S_i/S_0)$. Finally, when $(M/N)>(S_i/S_0)$, the obtained aperture field is different from the other cases.

This results can be summarized in the following conclusions:

- When $(M/N)<(S_i/S_0)$ the problem is accurately solved.
- In order to solve the problem efficiently the optimum ratio is $(M/N)=(S_i/S_0)$. This is the only ratio that also allows to solve accurately the problem when the direction of the indicent field is reversed. This optimum ratio agrees with that presented in reference (9).

Experimental and other numerical results.

In reference (5) experimental and numerical results are presented for the magnitude of reflection coefficient for WR-90 waveguide operated at 9

f=9 GHz. WR-90	Experimental (5) $\|\rho\|^2$	Modal Analysis A_2=10 A_1, B_2=10 B_1 M=36, N=900 $\|\rho\|^2$
water	0.72	0.72
chloroform	0.23	0.24

Table II Magnitude of reflection coefficient for WR-90 waveguide in contact with different medias.

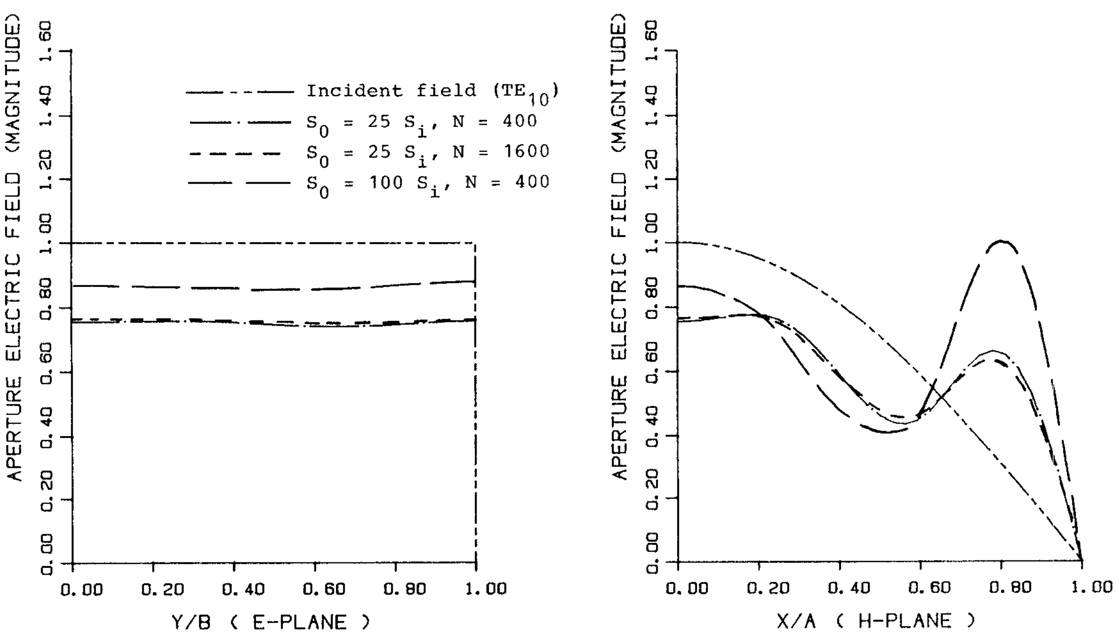

Fig. 3 Magnitude of the aperture electric field for different values of N and S_0. AxB=19x19mm. Frequency=2 GHz. Permittivity of the dielectric filling the input waveguide ε_r= 30; in half-space ε_r=46-j13.

GHz, in contact with water (ε_r=64-j30.5) and chloroform (ε_r=4.49-j0.85). A comparison between experimental results and our results is presented in table II, and a good agreement is observed.

HYBRID MODAL-SPECTRAL METHOD (HMSM)

In this method, the electromagnetic fields in the waveguide are expressed in the same way as in modal analysis, and in half-space they are expressed by the angular spectrum of plane waves (8). A similar procedure to that one used in modal analysis leads to the reflection matrix S_{11}.

A very similar method, called correlation matrix method, was proposed by R.H.Mc Phie et al(3). In this method, the following parameters will influence on the numerical solution:

- -Number of modes in the waveguide.
- -Number of samples used to evaluate the integrals containing the products of spectral functions for each pair of eigenmodes in the waveguide.

Numerical Results.

The S_{11} matrix for several rectangular apertures has been solved by HMSM. The problem was solved with several number of samples. In table III, the $S_{11}(1,1)$ of the square aperture already studied by modal analysis is shown.

Number of Samples	Muscle ε_r=46-j13 $\lvert\rho\rvert$	ϕ	fat ε_r=5.8-j.6 $\lvert\rho\rvert$	ϕ	ε_r=5.8-j.06 $\lvert\rho\rvert$	ϕ
9	.187	145.3	.6734	28.9	.696	26.3
17	.199	159.7	.6819	33.4	.707	30.8
33	.204	165.9	.6856	35.7	.712	33.1
65	.207	168.9	.6872	36.9	.714	34.3
129	.208	170.3	.6878	37.5	.715	34.9

Table III. TE_{10} reflection coefficient (magnitude $\lvert\rho\rvert$, phase ϕ) of a square aperture (AxB=19x19mm) for different lossy medias. Frequency=2 GHz. ε_r=30 in the waveguide.

From this table, it is interesting to remark that the convergence is obtained for a small number of samples (33x33).

Other reflection matrix coefficients do not change considerably when the number of samples is greater than 32x32. In that case there is no difference between the plots of the aperture field obtained with different number of samples. And also, these plots coincide with the ones obtained by modal analysis ($(M/N)\leqslant(S_i/S_0)$), Fig. 3.

COMPARISON AND CONCLUSION

Comparing the results obtained by both methods for lossy medias (biological tissues) it can be concluded that:

- i The reflection coefficients of the fundamental mode obtained by both methods are very similar. Other reflection matrix coefficients $S_{11}(i,j)$ obtained by both methods are close when $(M/N)\leqslant(S_i/S_0)$ and the imaginary waveguide is large enough.
- ii The transverse electric fields calculated in the aperture by both methods are very similar.
- iii The dimensions of the imaginary waveguide, used in modal analysis to simulate the half-space, must be increased when the losses decrease.
- iv For low-loss medias, the imaginary waveguide cross-section S_0, must be very large, and a great number of modes N should be considered. In these cases using HMSM increases accuracy and saves computer time.

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